Independent Design of Decentrallized Contollers for Multi Input Multi Output Processes

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Abstract
The main objective is to provide the system robustness and improved overall performance under significant model mismatches. In this a novel engineering oriented control system design methods for multivariable processes is presented. By employing the concepts of energy transmission ratio and effective relative gain, an Equivalent and Effective transfer function matrices for closed loop control system can be obtained. Consequently, the decentralized controllers can be independently designed by employing the single loop tuning techniques. These two design methods provide simple, straight forward and easy approaches for better computation. The real time experimentation carried out for the lower order MIMO process using VUDAQ-100 adds on card. The designed decentralized controllers are applied and the performances of both the methods have been analyzed.

Keywords: *Effective relative gain, energy transmission ratio, equivalent, effective transfer function, MIMO processes*

INTRODUCTION

Multi Input Multi Output processes are more difficult to deal compared to their Single Input Single Output counterparts due to the existence of interactions input and output variables. This topic has drawn a lot of research interests and many multivariable control approaches have been proposed. Among them, Model predictive control (MPC) has emerged as one of the most popular technique in process control industry [1, 2]. However, the computational complexity in presence of logical functions and mode switching make it more suitable on a higher level of the control system architecture to provide set points for regulation loops, while the PI/PID based control is still the dominant technique used at the lower level of the control systems. Most of the process control industries are using PI/PID type control loops. This is mainly attributed to its effectiveness and relatively simple structure, which can be easily understood and implemented in practice. Consequently, the research on PID control algorithm development and their applications is still a very active area; many formulae have been derived to tune the PID controllers over the years [3, 4].

Due to the high product quality and energy integration requirements, most of modern industry processes, however, are Multi-Input Multi-Output (MIMO) processes. For easier field implementation, it is desirable to apply well established single loop PID tuning principles to these MIMO processes. However, compared with Single-Input Single Multi-Output (SISO) counterparts, MIMO systems are more difficult to control due to the existence of interactions between input and output variables. Adjusting controller parameters of one loop affects the performance of the others, sometimes to the extent of destabilizing the entire system. To ensure stability, many industrial decentralized controllers are tuned loosely, which causes inefficient operation and higher energy costs [5, 6]. The equivalent transfer function can effectively approximate the dynamic interactions among loops. Consequently, the design of decentralized controller for MIMO processes can be converted to the design of single loop controllers. The method is simple, straightforward, easy to understand and implement. Several multivariable industrial processes with different interaction characteristics are employed to demonstrate the effectiveness and simplicity of the design method compared with the existing methods [7, 8].

PRELIMINARIES

An open loop stable multivariable system is considered with n inputs and n outputs, where r_i, i = 1,2,...,n, are the reference inputs; u_i, i = 1,2,...,n, are the manipulated variables; y_i, i = 1,2,...,n, are the system outputs,

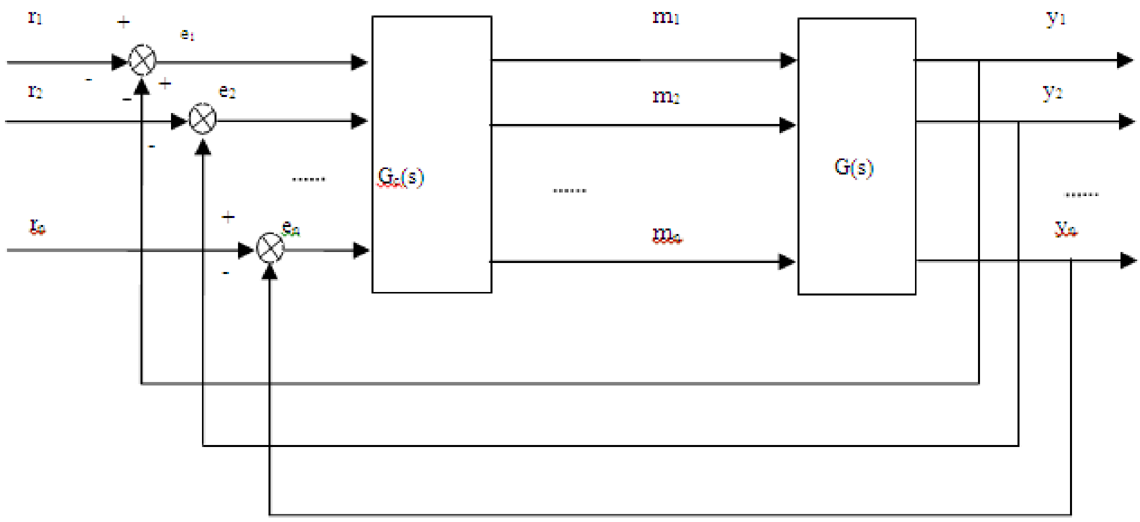

Fig. 1: *Closed-Loop Multivariable Control System.*

The process transfer function matrix G(s) and decentralized controller matrix $G_c(s)$ with compatible dimensions are expressed by

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \dots & g_{2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s) & g_{n2}(s) & \dots & g_{nn}(s) \end{bmatrix}$$

and

$$G_c(s) = \begin{bmatrix} g_{C1}(s) & 0 & \dots & 0 \\ 0 & g_{C2}(s) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & g_{cn}(s) \end{bmatrix}$$

Let $g_{ji}(j\omega) = K_{ij} g_{ij}^0(j\omega)$,

Where k_{ij} and $g_{ij}^0(j\omega)$ are steady state gain and normalized transfer function of $g_{ij}^0(j\omega)$ *i.e.* $g_{ij}^0(j\omega)=1$, respectively. The interaction among individual loop is described by ERGA, the main result of ERGA is summarized as follows. Define e_{ij} of a particular transfer function as

$$e_{ij} = k_{ij} \int_0^{\omega} c_{ij} \left| g_{ij}^0(j\omega j \right| d\omega$$

where $\omega_{c,ij}$ for ri,j =1,2,...,n are the critical frequency of the transfer function gij (jω) and |•| is the absolute value of •. In order to calculate e_{ij}, the critical frequency can be defined in two ways:

1. $\omega_{c,ij} = \omega_{B,ij}$ where $\omega_{B,ij}$ for i,j = 1,2,…,n is the band width of the transfer function $g_{ij}^{0}(j\omega)$ and determined by the frequency where the magnitude plot of frequency response reduced to 0.707 time, i.e., $|g_{ij}(j\omega_{B,ij})| = 0.707|g_{ij}(0)|$.
2. $\omega_{c,ij} = \omega_{u,ij}$, where $\omega_{u,ij}$ for i,j = 1,2,...,n is the ultimate frequency of the transfer function $g_{ij}^{0}(j\omega)$ and determined by the frequency where the phase plot of frequency response across -π, i.e.,

   Express the energy transmission ratio array as

$$\mathbf{E} = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots & e_{nn} \end{bmatrix}$$

To simplify the calculation the effective energy transmission ratio array is given as:

$$\mathbf{E} = \mathbf{G}(0) \otimes \mathbf{\Omega},$$

Where the operator ⊗ is the Hadamard product,

$$\mathbf{G}(0) = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} \text{ and}$$

$$\mathbf{\Omega} = \begin{bmatrix} \omega_{u11} & \omega_{12} & \cdots & \omega_{1n} \\ \omega_{21} & \omega_{22} & \cdots & \omega_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \omega_{u,n1} & \omega_{n2} & \cdots & \omega_{nn} \end{bmatrix}$$

are the steady state gain array and the critical frequency array, respectively. Since e_{ij} is an indication of energy transmission ratio for loop y_i-u_j, the bigger the e_{ij} value is, the more dominant of the loop will be.

The effective relative gain, ϕ_{ij}, between output variable y_i and input variable u_j is define as the ratio of two effective energy transmission ratio :

$$\phi_{ij} = \frac{e_{ij}}{\hat{e}_{ij}} \qquad (1)$$

When the effective relative gains are calculated for all the input/output combinations of a multivariable process, it results in an array, ERGA, which can be calculated by

$$\Phi = E \otimes E^{-T} = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}$$

The ERGA is used to determine the best variable paring. Suppose that the best loop configuration has been determined and the best pair is diagonally placed in the transfer function matrix. Similar to the open loop gain, we let the effective energy transmission ratio, $\hat{e}_{ij}$, when all other loops are closed be, $\hat{e}_{ij}(0) = \hat{g}_{ij}(0)\hat{\omega}_{u,ij}$ i,j =1,2,...,n, where $\hat{g}_{ij}$ and $\hat{\omega}_{u,ij}$ are the steady state gain and ultimate frequency between output variable y_i and input variable u_j when all other loops are closed, respectively. Then, from Eq. (1)

$$\hat{g}_{ij}(0)\ \hat{\omega}_{u,ij} = \frac{g_{ij}(0)\omega_{u,ij}}{\phi_{ij}} \qquad (2)$$

By the definition of RGA, we have

$$\hat{g}_{ij}(0) = \frac{g_{ij}(0)}{\lambda_{ij}} \qquad (3)$$

Where λ_{ij} is the relative gain.

Substitute Eq. (3) into (2) and rearrange to result

$$\frac{\phi_{ij}}{\lambda_{ij}} = \frac{\omega_{u,ij}}{\hat{\omega}_{u,ij}} \equiv \gamma_{ij,} \qquad (4)$$

where γ_{ij} represents the critical frequency change of loop *i-j* when other loops are closed, defined as relative critical frequency. When the relative frequencies are calculated for all the input/output combinations of a multivariable process, it results in an array, i.e., relative frequency array (RFA).

We can let the ETF have the same structures as the corresponding open loop transfer functions but with different parameters

$$\hat{g}_{ii}(s) = \hat{g}_{ii}(0)\ g^{r}_{ii}(s)\ e^{-\hat{d}_{ii}s}, \quad (5)$$

and $\hat{d}_{ii}$ is the time delay of the ETF. $\hat{d}_{ii}$ can be determined from Eq (4).

$$\hat{d}_{ii} \approx \frac{\omega_{u,ij}}{\hat{\omega}_{u,ij}} d_{ii} = \gamma_{ii} d_{ii} \quad (6)$$

This is the practical formula which will be used to derive the Equivalent and Effective transfer functions.

DECENTRALLIZED CONTROLLER DESIGN APPROACHES

Without loss of generality, we assume that each main loop, i.e., diagonal element in the transfer function matrix is represented by a second order plus dead time (SOPDT) model, which can be used to describe most of the industrial processes [9, 10].

The PID controller of each loop is supposed of the following standard form.

$$g_{c,ji}(s) = k_{p,ji} \frac{1}{k_{i,ji}s} + k_{d,ji}s \quad (7)$$

The controller can be rewritten as

$$g_{c,ji}(s) = k_{ji} \frac{As^2 + Bs + C}{s}$$

Where $A = k_{d,ji}/k_{ji}$, $B = k_{p,ji}/k_{ji}$ and $c = k_{i,ji}/k_{ji}$

By selecting $A=a_{2,ij}$, $B=a_{1,ij}$ and c=1, the forward transfer functions are calculated functions

$$g_{c,ji}(s)\hat{g}_{ij}(s) = k_{ji} \frac{\hat{g}_{ij}(0)}{s} e^{-\hat{l}_{ij}s}$$

Denoting the gain and phase margin specifications as $A_{m,ij}$ and $\Psi_{m,ij}$ and their crossover frequencies as $\omega_{g,ij}$ and $\omega_{p,ij}$ respectively, we have

$$\arg[g_{c,ji}(j\omega_{g,ij})\hat{g}_{ij}(j\omega_{g,ij})] = -\pi,$$

$$A_{m,ij}[g_{c,ji}(j\omega_{g,ij})\hat{g}_{ij}(j\omega_{g,ij})] = 1$$

$$[g_{c,ji}(j\omega_{p,ij})\hat{g}_{ij}(j\omega_{p,ij})] = 1$$

$$\psi_{m,ij} = \pi + \arg[g_{c,ji}(j\omega_{p,ij})\hat{g}_{ij}(j\omega_{p,ij})]$$

By substitution and simplification to above equations, we obtain $\omega_{g,ij}\hat{I}_{ij} = \frac{\pi}{2}$, $A_{m,ij} = \frac{\omega_{g,ij}}{k_{ji}\hat{g}_{ij}(0)}$

$$k_{ji}\hat{g}_{ij}(0) = \omega_{p,ij}, \qquad \psi_{m,ij} = \frac{\pi}{2} - \omega_{p,ij}I_{ij}$$

Which results

$$\psi_{m,ij} = \frac{\pi}{2}\left(1 - \frac{1}{A_{m,ij}}\right), \quad k_{ji} = \frac{\pi}{2A_{m,ij}\hat{I}_{ij}\hat{g}_{ij}(0)}$$

By this formulation, the gain and phase margin are interrelated to each other, some possible gain and phase margin selections are given in Table 1.

Table 1: *Typical Gain and Phase Margin Values.*

$\psi_{m,i}$	π/4	π/3	3π/8	2π/5
$A_{m,i}$	2	3	4	5

Equivalent Transfer Function Method

Each controller can thus be independently designed by single loop approaches based on the corresponding diagonal transfer functions. The equivalent transfer function can be represented by:

$$g_{ii}(s) = \frac{g_{ii}(0)}{a_{2,ii}s^2 + a_{1,ii}s + 1} e^{-I_{ii,s}} \quad (8)$$

based on the Eq. (3) the PID parameters are given by

$$\begin{bmatrix} k_{p,ji} \\ k_{i,ji} \\ k_{d,ji} \end{bmatrix} = \frac{\pi\lambda_{ij}}{2A_{m,ij}\gamma_{ij}I_{ij}g_{ij}(0)} \begin{bmatrix} a_{2,ij} \\ 1 \\ a_{1,ij} \end{bmatrix} \quad (9)$$

In addition, the control performance may not be met well for some high dimensional systems since a fixed PI/PD structure is adopted.

The design of full dimensional PI/PD controller consists of two parts.

1) Off-diagonal controllers: The main task of the off-diagonal controllers is to minimize the interactions among loops.
2) Diagonal controllers: The diagonal controllers are to provide the desired performance of the closed loop control system.

For off-diagonal elements, according to Eq. (1)

$$sg_{c,ji}(s) = \frac{1}{\hat{g}_{c,ji}(s)} \Rightarrow g_{c,ji}(s)\hat{g}_{ji}(s) = 1/s \quad (10)$$

Image that $g_{c,ji}(s)\hat{g}_{ij}(s)$ is the forward transfer function of an artificial closed loop control system, the control object $g_{c,ji}(s)\hat{g}_{ij}(s) = 1/s$, is to obtain an ideal control for this loop. However, since the perfect control cannot be realized in practice, we use the gain and phase margins approach to design the controller.

Effective Transfer Function Method

The Effective transfer function is represented as:

$$\hat{g}_{ij}(s) = \frac{\hat{g}_{ii}(0)}{a_{2,ii}s + a_{i,ii}s + 1} e^{-diis} \quad (11)$$

For the four different combinations of λii and γii, $\hat{g}_{ij}(s)$ may take different modes are shown in Table 2.

Case 1: $\lambda_{ii} \leq 1$, $\gamma_{ii} \leq 1$
Case 2: $\lambda_{ii} \leq 1$, $\gamma_{ii} > 1$
Case 3: $\lambda_{ii} > 1$, $\gamma_{ii} \leq 1$
Case 4: $\lambda_{ii} > 1$, $\gamma_{ii} > 1$

The corresponding PID parameters are given by

$$\begin{bmatrix} k_{p,ii} \\ k_{i,ii} \\ k_{d,ii} \end{bmatrix} = \frac{\pi}{2A_{m,ii}I_{ii}g_{ii}(0)} \begin{bmatrix} a_{1,ii} \\ 1 \\ a_{2,ii} \end{bmatrix}$$

(12)

***Table 2:** Decentralized PID Controller Design.*

Mode	$\hat{g}_{ii}$ (s)	$k_{p,ii}$	$k_{i,ii}$	$k_{d,ii}$
$\lambda_{ii} \leq 1, \gamma_{ii} \leq 1$	$\frac{g_{ii}(0)/\lambda_{ii}}{a_{2,ii}s^2 + a_{1,ii}s + 1} e^{-diis}$	$\frac{\pi\lambda_{ii}a_{1,ii}}{2A_{m,i}d_{ii}g_{ii}(0)}$	$\frac{\pi\lambda_{ii}}{2A_{m,i}d_{ii}g_{ii}(0)}$	$\frac{\pi\lambda_{ii}a_{2,ii}}{2A_{m,i}d_{ii}g_{ii}(0)}$
$\lambda_{ii} \leq 1, \gamma_{ii} > 1$	$\frac{g_{ii}(0)/\lambda_{ii}}{a_{2,ii}s^2 + a_{1,ii}s + 1} e^{-\gamma_{ii}d_{ii}s}$	$\frac{\pi\lambda_{ii}a_{1,ii}}{2A_{m,i}\gamma_{ii}d_{ii}g_{ii}(0)}$	$\frac{\pi\lambda_{ii}}{2A_{m,i}\gamma_{ii}d_{ii}g_{ii}(0)}$	$\frac{\pi\lambda_{ii}a_{2,ii}}{2A_{m,i}\gamma_{ii}d_{ii}g_{ii}(0)}$
$\lambda_{ii} > 1, \gamma_{ii} \leq 1$	$\frac{g_{ii}(0)}{a_{2,ii}s^2 + a_{1,ii}s + 1} e^{-diis}$	$\frac{\pi a_{1,ii}}{2A_{m,i}d_{ii}g_{ii}(0)}$	$\frac{\pi}{2A_{m,i}d_{ii}g_{ii}(0)}$	$\frac{\pi a_{2,ii}}{2A_{m,i}d_{ii}g_{ii}(0)}$
$\lambda_{ii} > 1, \gamma_{ii} > 1$	$\frac{g_{ii}(0)}{a_{2,ii}s^2 + a_{1,ii}s + 1} e^{-\gamma_{ii}d_{ii}s}$	$\frac{\pi a_{1,ii}}{2A_{m,i}\gamma_{ii}d_{ii}g_{ii}(0)}$	$\frac{\pi\lambda_{ii}}{2A_{m,i}\gamma_{ii}d_{ii}g_{ii}(0)}$	$\frac{\pi a_{2,ii}}{2A_{m,i}\gamma_{ii}d_{ii}g_{ii}(0)}$

SYSTEM DESCRIPTION AND REAL TIME EXPERIMENTATION

The dynamic non-linear process considered under study is an interacting two tank system shown in Figure 1. The level of tank 1 and tank 2 are chosen as h1, h2. The manipulated variables are chosen as inflow of tank 1 (fin 1) and inflow tank 2 (fin 2).

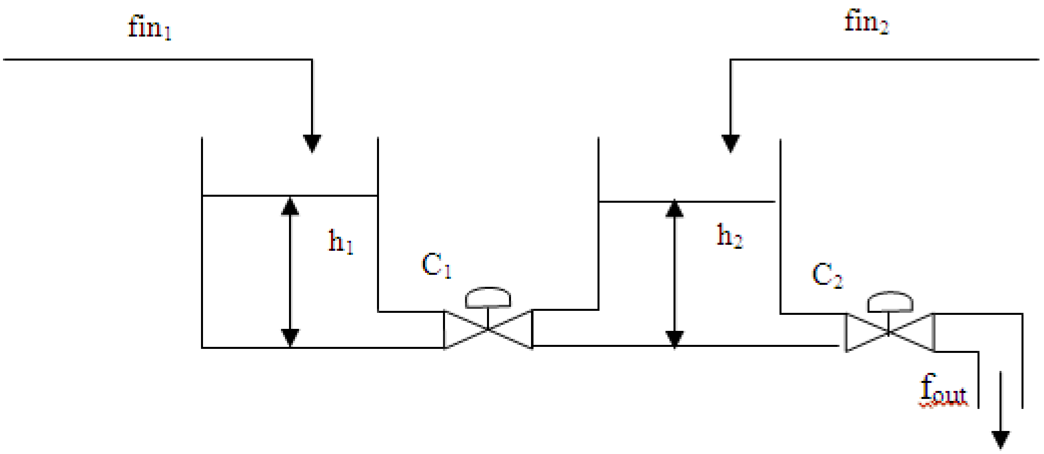

Fig. 2: *Two Tank Interacting System.*

The material balance for the two tank system yields the following equations and the steady state operating data of the two tank system are given in Table 3.

Table 3: *Steady State Operating Data of Two Tank System.*

Area of the tank A_T	$0.0154m^2$
Constant Outflow coefficient (C_1)	1
Constant Outflow Coefficient (C_2)	0.8
Acceleration due to gravity(g)	$9.80m/sec^2$

$$\frac{dh_1}{dt}=\frac{fin_1}{A_T}-\frac{C_1S_p\sqrt{2g(h_1-h_2)}}{A_T} \quad (13)$$

$$\frac{dh_2}{dt}=\frac{C_1S_p\sqrt{2g(h_1-h_2)}}{A_T} \quad (14)$$

Where

C_1, C_2 → constant flow coefficient

S_p → cross section of the connecting pipe

g → acceleration due to gravity

A_T → Area of the tank

Determination of Model

The steady state of the two tanks is 50ml. After reaching the level set point change of 10ml is given to tank 1 the corresponding tank 1 and tank +2 responses have taken, similarly the same set point change given to tank 2 the responses have taken and shown in Figures 3 and 4.

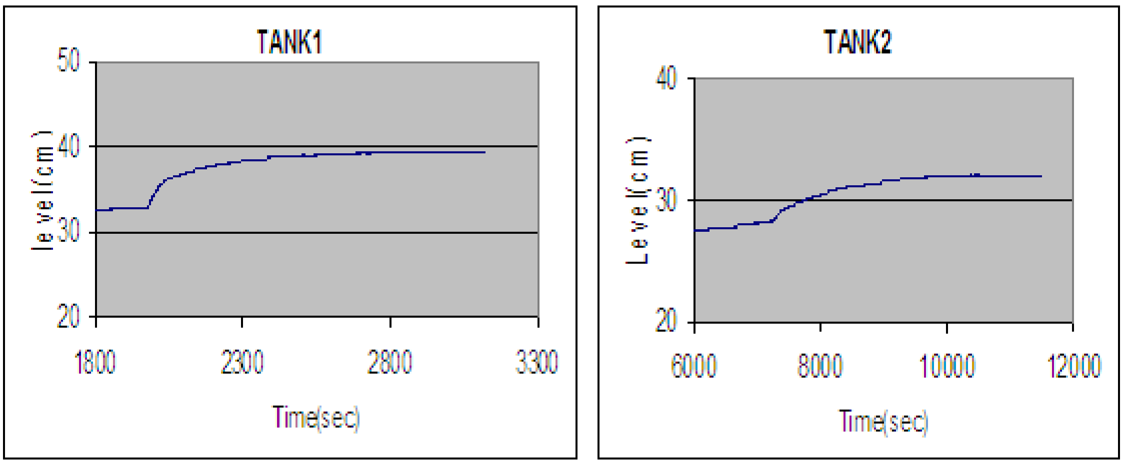

Fig. 3: *Change in Inflow of 10ml /sec given to Tank - 1 at 2,000 (sec).*

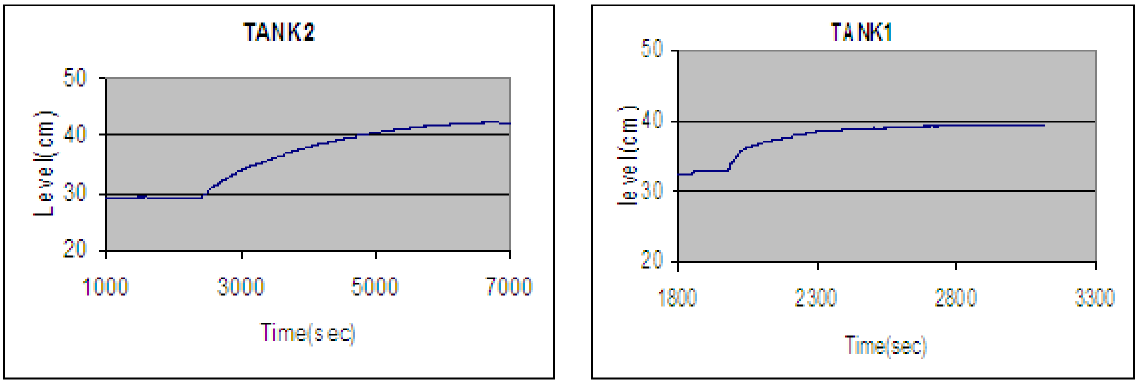

***Fig. 4:** Change in Inflow of 10ml/sec given to Tank - 2 at 2,000 (sec).*

The determined real time process is given below

$$G(s)=\begin{bmatrix} \frac{2.5}{60s+1}e^{-40s} & \frac{2.0}{60s+1}e^{-20s} \\ \frac{3.5}{97.5s+1}e^{-17.5s} & \frac{3.3}{127.5s+1}e^{-15.5s} \end{bmatrix}$$

The RGA, critical frequency array, ERGA and RFA are given respectively by

$$\Lambda=\begin{bmatrix} 2.3042 & -3.2041 \\ -9.8324 & 2.3042 \end{bmatrix}$$

$$\Omega=\begin{bmatrix} 0.0165 & 0.0215 \\ 0.0250 & 0.0150 \end{bmatrix}$$

$$\Phi=\begin{bmatrix} 3.0842 & -1.2075 \\ -4.0432 & 3.0842 \end{bmatrix}$$

$$\Gamma=\begin{bmatrix} 0.1976 & 0.0876 \\ 0.0957 & 0.2140 \end{bmatrix}$$

where
Λ = Relative gain array
Ω = Critical frequency array
Φ = Effective relative gain array
Γ = Relative frequency array

Equivalent Transfer Function Method
To validate the time delays of the transfer function the original time delays are to be calculated

$$G_c(s)=\begin{bmatrix} 1.4608\left(1+\frac{1}{40s}\right) & 0.3787\left(1+\frac{1}{20s}\right) \\ -0.5237\left(1+\frac{1}{17.5s}\right) & 1.5823\left(1+\frac{1}{15.5s}\right) \end{bmatrix}$$

Effective Transfer Function Method
Following ERGA indicate diagonal pairing, i.e., 1-1/2-2. As $\lambda_{ii}>1$ and $\gamma_{ii}<1$, referring Table 2 the proposed method equivalent process for two loop are calculated as $\frac{2.5}{60s+1}e^{-40s}$ and $\frac{3.3}{127.5s+1}e^{-15.5s}$ respectively. For the gain and phase margins of 2 and $\pi/4$,

The real time results for Equivalent and Effective transfer function methods of servo and regulatory responses with set point change given to both tank-1 and tank -2 are shown in Figures 5–8.

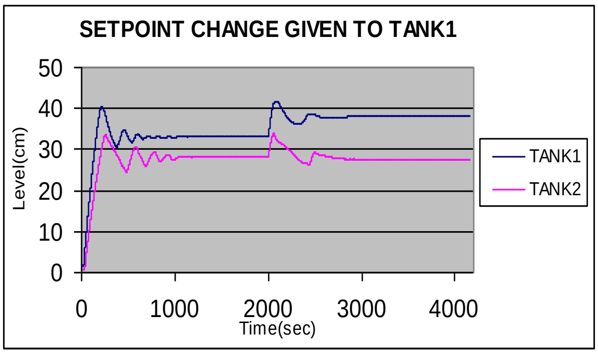

***Fig. 5:** Servo and Regulatory Response of Equivalent Transfer Function Method for a Set Point Change from 33cm to 38cm given at t=2,000 (sec).*

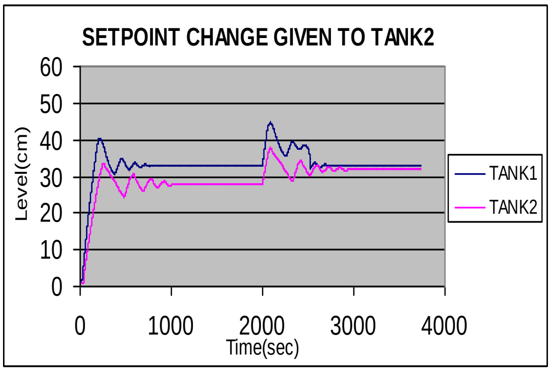

Fig. 6: *Servo and Regulatory Response of Equivalent Transfer Function Method for a Set Point Change from 28cm to 32cm given at t=2,000 (sec).*

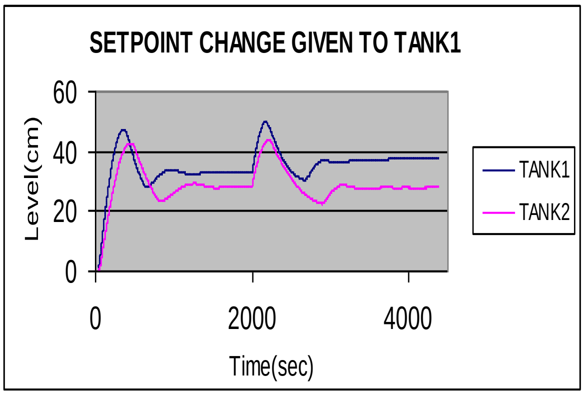

Fig. 7: *Servo and Regulatory Response of Effective Transfer Function Method for a Set Point Change from 33cm to 38cm given at t=2,000 (sec).*

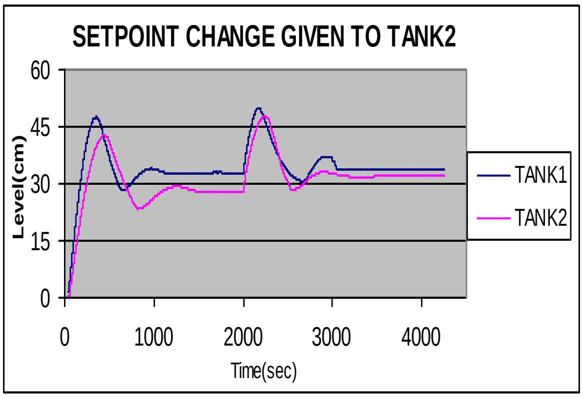

Fig. 8: *Servo and Regulatory Response of Effective Transfer Function Method for a Set Point Change from 28cm to 32cm given at t=2,000 (sec).*

Table 4: *Comparative Performances of Two Methods.*

S. No.	Parameters	Loop 1		Loop 2	
		ETF1	**ETF2**	**ETF1**	**ETF2**
1	Settling Time (sec)	2800	3800	2500	3000
2	Peak Time (sec)	2100	2200	2100	2300
3	Peak Over Shoot (%)	42	50	44	49
4	Integral Square Error	1.134	1.543	1.025	1.678

CONCLUSION

This paper presents novel methods to construct Equivalent and Effective transfer function models for decentralized control system design of multivariable interactive processes. The simplicity and effectiveness of the models is based on the energy transmission ratio of each individual transfer function which provides necessary information of gain and frequency changes when all other loops are closed. Consequently, the decentralized controllers can be obtained by simply using single loop design approaches.

The real time results for 2x2 processes shows that the Equivalent Transfer Function method provides overall better performance and provides the advantage of robustness it can still work with satisfactory performance even under significant model mismatches compared to Effective Transfer Function method. The Equivalent Transfer Function method even more significant when applied to higher dimensional processes with complicated interaction modes.

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